

義守大學九十二學年度轉學生入學招生考試

『線性代數』參考試題

1. Enlarge the independent set $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} \right\}$ to a basis for R^3 . (10%)

2. Let $A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 3 & 6 & 1 & 11 \\ 2 & 4 & 1 & 8 \end{bmatrix}$

- (1). Find the rank of A. (5%)
- (2). Find the nullspace of A. (10%)

3. Let T be a linear transformation $T : R^3 \rightarrow R^3$ defined by $T \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 - 2x_2 + x_3 \\ x_2 - x_3 \\ 2x_2 - 3x_3 \end{bmatrix}$.

- (a). Find the standard matrix representation of T . (5%)
- (b). Is T invertible? If yes, find a formula for T^{-1} . (10%)

4. Let $W = sp\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ be a subspace of R^3 .

- (a). Find an orthonormal basis for the subspace W . (10%)
- (b). Find the projection matrix for the subspace W . (10%)

5. If $A \in R^{n \times n}$ satisfies $x^T A x > 0$ for all nonzero vector x . Show that all eigenvalues of A are positive. (10%)

6. Diagonalize the matrix $A = \begin{bmatrix} 1 & -3 & 3 \\ 0 & -5 & 6 \\ 0 & -3 & 4 \end{bmatrix}$ if possible. (10%)

7. Let A be an $m \times n$ matrix of rank r . Show that the $n \times n$ symmetric matrix $A^T A$ also has rank r . (10%)

8. Prove that if A is a Hermitian matrix, then there exists a unitary matrix U such that $U^{-1} A U$ is a diagonal matrix. (10%)