※此為試題卷，請將答案填寫在答案卷內，未寫於答案卷內者，不予計分。

※不可使用計算機

1. Please find the general solution of the following equation. (20%)

$$\ddot{y} - 3\dot{y} + 2y = xe^x$$

2. Please find the general solution of the following equation. (20%)

$$x^3\dddot{y} - 2x^2\ddot{y} + 5x\dot{y} - 5y = 0$$

3. Solve the problem with initial condition. (20%)

$$\ddot{y} - 4\dot{y} + 3y = 10e^{-2t}, \quad y(0) = 1, \quad \dot{y}(0) = -3$$

4. Let the given matrix be $\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Please find the matrices $\mathbf{Q}$ and $\mathbf{D}$

   such that $\mathbf{Q}^{-1}\mathbf{A}\mathbf{Q} = \mathbf{D}$. (20%)

5. Please find the eigenvalue and the eigenvector for the 3×3 matrix (20%)

$$\mathbf{A} = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$